

# Statistics of energy spectra of a strongly disordered system of interacting electrons

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## Abstract

Statistics of many particle energy levels of a finite two-dimensional system of interacting electrons is numerically studied. It is shown that the statistics of these levels undergoes a Poisson to Wigner crossover as the strength of the disorder is decreased. This transition occurs at a similar strength of disorder as the one-electron delocalization crossover in a finite 2d system and develops almost simultaneously at all energies. We interpret this crossover in terms of delocalization in the space of occupation numbers of strongly bound and compact electron-hole pairs (excitons).

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## I. INTRODUCTION

The statistics of the quantum energy spectra of a disordered system of noninteracting electrons, for example, the Anderson model, was shown to be a good diagnostic tool to study an insulator-metal transition [1]. It was discovered that the crossover from Poisson distribution of the nearest neighbor level spacings to a Wigner distribution sharpens with the system size. Finite size scaling then permits to find out whether the transition exists and if it does to calculate quite accurately transition point and indexes [1,2].

Recently attention has started to shift in the direction of spectral statistics of the total energy of a finite disordered system of interacting electrons [3–8]. Good examples of such systems are quantum dots (here we are talking about the energies of the excited states of the dot and not about the charging spectrum). All previous works that we know of deal with metallic systems which are well above insulator-metal transition. For such system a Poisson-Wigner (P-W) crossover was predicted with a growing energy or interaction strength [3–6,8]. The relation of this crossover to delocalization in Fock space and the decay of one electron states was discussed [3]. The intriguing question of whether such statistics can work as a tool to study insulator-metal transition has not yet been addressed. In this paper we concentrate on the two-dimensional case where the very existence of a transition in an interacting system is under debate for a long time.

We started this work by exploring whether we can obtain any evidence for the transition by studying the finite size effects in the statistics of levels of the total energy of a many particle system. As far as we know this is the first study of such statistics at the insulating side of the delocalization crossover and at the crossover itself. We did not find any conclusive evidence for this crossover. Instead we found that the delocalization crossover as function of the decreasing disorder generates a P-W crossover in the excited state statistics which takes place almost uniformly at all energies larger than the single level spacing (the energy of the many body excited states are calculated from the many body ground state). We interpret the excited states with high energies as consisting of several electron-hole excitations. Each

electron-hole pair is bound by Coulomb interaction. We call such excitations excitons. At strong enough disorder states are localized in the space of states with different number of excitons or, in other words, states of very close energies but with different number of excitons do not mix coherently. As a result, the nearest level distribution function of many body states is Poissonian. With decreasing disorder the rate of decay of an exciton to smaller energy excitons becomes of the order of spacing between many particle levels. The fact that this P-W crossover is almost independent on energy and interaction differs drastically from what happens in metallic samples [3–6,8]. This occurs because the density of states of the excitons and the exciton-exciton interactions are different from the ones for weakly interacting electron-hole pairs in a metal.

## II. MODEL

The numerical study is based on the following interacting many-particle tight-binding Hamiltonian:

$$H = \sum_{k,j} \epsilon_{k,j} a_{k,j}^\dagger a_{k,j} - V \sum_{k,j} (a_{k,j+1}^\dagger a_{k,j} + a_{k+1,j}^\dagger a_{k,j}) + h.c + H_{int}, \quad (1)$$

where  $\epsilon_{k,j}$  is the energy of a site  $(k, j)$ , chosen randomly between  $-W/2$  and  $W/2$  with uniform probability, and  $V$  is a constant hopping matrix element. The interaction Hamiltonian is given by:

$$H_{int} = U \sum_{k,j>l,p} \frac{a_{k,j}^\dagger a_{k,j} a_{l,p}^\dagger a_{l,p}}{|\vec{r}_{k,j} - \vec{r}_{l,p}|/b} \quad (2)$$

where  $U = e^2/b$  and  $b$  is the lattice unit.

We consider  $3 \times 3$ ,  $4 \times 3$ , and  $4 \times 4$  dots with  $m = 9, 12, 16$  sites and  $n = 3, 4, 4$  electrons. The  $M \times M$  Hamiltonian (where  $M = \binom{m}{n}$ ) matrix is numerically diagonalized and all the eigenvectors  $|\Psi_j\rangle$  and eigenvalues  $E_j$  are obtained. The strength  $U$  of the interaction is varied between  $0 - 30V$ . and the disorder strength is chosen to be between  $W = 5V$  and  $W = 100V$ . Usually results are averaged over 1000 realizations.

We will use the energy level statistics as an indication of the Anderson transition in Fock space. A convenient way to characterize the change in the statistics of a system proposed in Ref. [1] is to study the parameter  $\gamma$  defined as

$$\gamma = \frac{\int_2^\infty P(s)ds - e^{-\pi}}{e^{-2} - e^{-\pi}}, \quad (3)$$

where  $P(s)$  is the distribution of the normalized level spacings  $s = E_j - E_{j-1}/\langle E_j - E_{j-1} \rangle$ , where  $\langle \dots \rangle$  denotes an average over different realizations of disorder. For an infinite system  $\gamma$  changes sharply from  $\gamma = 1$  in the localized regime to  $\gamma = 0$  in the extended regime. For a finite system the change is gradual.

### III. RESULTS AND DISCUSSION

We begin by presenting the behavior of  $\gamma$  for the non-interacting spacing between the many particle ground state and the first excited state. A crossover from a Wigner to Poisson statistics as  $W$  increases is clearly seen (Fig. 1).

From finite size studies of one electron problem one can estimate that the point for which the localization length  $\xi$  is of the system size corresponds to  $\gamma = 0.35$  [1]. In our case such values of  $\gamma$  are realized at  $W \sim 15V$ . This does not contradict to the calculation by MacKinnon and Kramer of  $\xi(W = 15) = 2.2b$  [9]. So the P-W crossover for the first excitation energy happens close to the finite size delocalization crossover in noninteracting system, which is expected.

In the inset of Fig. 1 we show the distribution of the level spacings close to the many particle band center for  $U = 10V$ . A clear crossover from a Wigner like behavior to a Poisson behavior can be seen. In order to show this behavior for different excitation energies and interaction strength we present in Fig. 2 gray scale maps of  $\gamma$  for three different values of  $W$ . The gray scale maps shows the average value of  $\gamma$  for the spacings between a many particle state with energy  $E$  above the ground state and the many particle state above it. A general feature which appears with growing disorder strength at  $W \sim 15$  is apparent - the statistics

for energies  $E > 2\Delta$  (where  $\Delta$  is the single electron level spacing) is rather uniform and does not depend strongly on interaction strength nor on excitation energy. This feature becomes more pronounced as the disorder increases and is in strong contrast to the situation in the metallic regime, in which interesting features were seen as function of the interaction strength and excitation energy.

This behavior clearly shows that at a large  $W$  the different high energy many particle states can be close in energy but nevertheless can have small repulsion, i.e., interactions do not couple different many particle states no matter what energy is available. We interpret this behavior as the result of the high energy many particle states being composed of several electron-hole excitations (excitons). Neighboring many particle states usually are composed of a different number of excitons and are related by a very weak interaction matrix element between them no matter how strong the interactions are. Thus, no repulsion between the states appears and the statistics is essentially Poissonic for any interaction strength or energy. The P-W crossover as the disorder  $W$  decreases is rather uniform and shows no strong dependence on energy or interaction strength (as long as one is still above the transition and the interaction  $U > 2V$ ). We interpret this crossover as result of the delocalization of the system in the space of states with different number of excitons which happens once the matrix element for the decay of a typical exciton into two smaller ones becomes of the order of the spacing between many particle levels.

The energy independent P-W crossover revealed here differs drastically from the predictions and calculations made for Fock space delocalization in metallic systems [3–8]. We relate this fact to the difference between the excitons of the insulating phase and the weakly interacting electron-hole pairs of the metallic samples. Contrary to the latter ones, excitons consist of an electron and a hole strongly bounded to each other by the Coulomb interaction. In the limit of large  $W$  and  $U$  in the classical Coulomb glass this exciton is just the classical compact electron-hole pair excitation of Ref. [10]. Due to the existence of the Coulomb gap, electron-hole excitations corresponding to the transfer of an electron by a small distance (compact pair) are known to have a constant density of states at small energies. On the

other hand, in a metallic dot the joint density of states of weakly interacting electron-hole pairs is linear in energy. We expect that such a difference is preserved in the quantum system. To find the P-W crossover we have to use the density of states of excitons and the matrix elements for an exciton decay. We have already mentioned that there is a drastic difference between the exciton density of states of metallic and insulating samples. The matrix elements of the exciton decay should be different from the matrix element of the emission of an electron-hole pair by a free electron used in Refs. [3–7] as well.

Thus a drastic difference between the P-W crossovers for metallic and insulating cases seems to be natural. Unfortunately we could not prove the uniform energy P-W crossover in the insulating case.

In Fig. 3 we present a more quantitative description of the P-W crossover. We show the results for  $\gamma$  averaged over 3% and 10% of the low lying many particle energy levels for intermediate values of the interaction ( $U = 8V, 12V, 16V$ ) and for different lattice sizes and electron numbers. It is obvious that in all cases  $\gamma$  increases as the disorder  $W$  is enhanced. This is a possible indication of the signature of the single electron delocalization crossover on the many particle spectrum. There is no significant difference between the values of  $\gamma$  for 3% and 10% of the spectrum, nor a strong dependence on interaction strength, so that crossover indeed happens rather uniformly in energy and interaction strength. As larger values of disorder are approached the difference becomes even smaller and the many particle spectrum becomes even more uniform.

Because not much depends on energy there is an a priori chance that this crossover somehow reflects an insulator-metal transition in a many particle interacting system. As can be seen in Fig. 3 there is no clear finite size behavior. This probably means that we are dealing with a crossover, not a phase transition.

Before we conclude we want to comment on the importance of excitons introduced above. Now we concern ourselves with the localization of excitons in real space. The following scenario seems likely (although we have not found a way to prove it using our numerical data). Excitons are localized as long as the disorder dominates and the many body level statistics

is Poissonian. There might exist a crossover regime where the charge is still localized but the excitons become delocalized. In the case when a metal-insulator transition exists (in three and possibly for the interacting case in two dimensions) the exciton delocalization happens on the insulating side of the critical region of the metal-insulator transition. This scenario can result in a situation where electronic conductivity is exponentially small while the electronic thermal conductivity changes as a power of temperature [11].

Another consequence of the possible exciton delocalization is that they can play a crucial role in low temperature variable range hopping. At low temperatures they can assist electron hopping much more effectively than phonons. As a result the prefactor of the variable range hopping can acquire an universal value  $e^2/h$ , which was observed experimentally. This in turn leads to a very simple microscopic interpretation of the dynamic scaling at a number of quantum phase transition points, such as the quantum Hall or the superconductor-insulator transitions [12,13].

Arguments for the delocalization of two interacting electrons above the Fermi sea in a situation where both of them are localized (as well as the other electrons of the Fermi sea) were given in Refs. [14,15]. In the case of electrons interacting via the Coulomb interaction these arguments should not work because the joint density of states of two electrons drastically decreases at small energy due to the Coulomb gap in the one-electron density of states. However, for a compact electron-hole pair, an exciton, as we mentioned above, Coulomb effects increase its density of states making these arguments more plausible. Actually these arguments were applied to the exciton before [15], however, the effect of the Coulomb enhancement was not considered.

In conclusion, we found a P-W crossover in the statistics of the nearest neighbor spacings of many particle levels, which occurs almost simultaneously at all energies. To interpret this crossover we introduced excitons and speculated that the crossover is related to the increase of their interaction which in turn leads to the transition from a description where each state corresponds to a number of weakly interacting excitons to new states which are delocalized in the space of the old ones. This transition is similar to delocalization in Fock space in

metallic samples recently studied in Refs. [3–8].

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# FIGURES

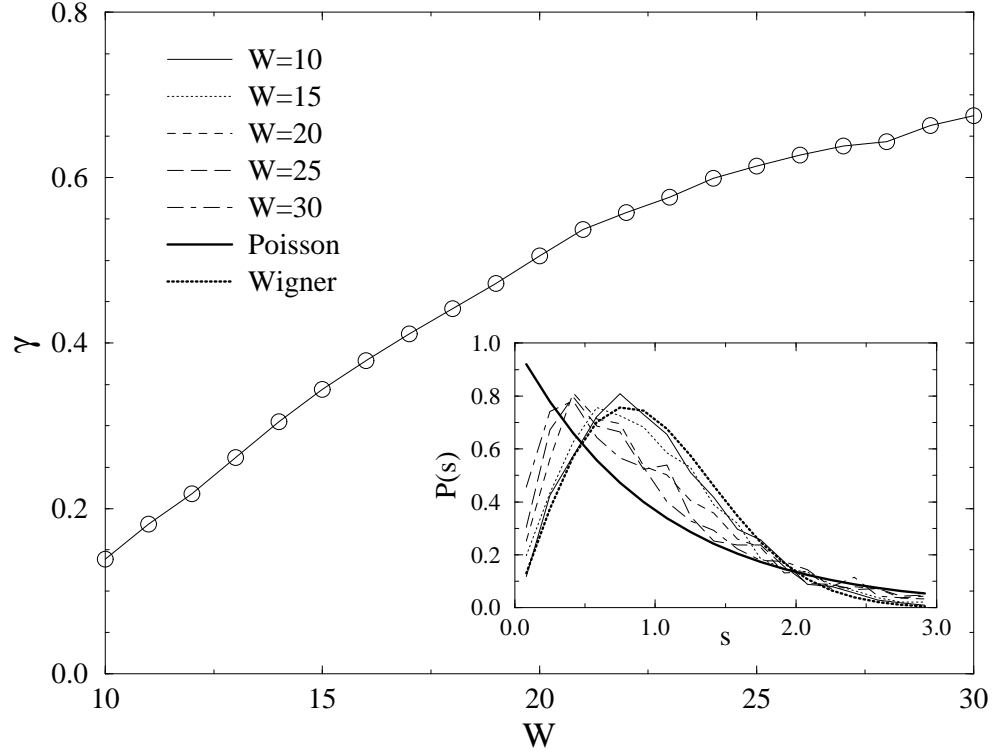
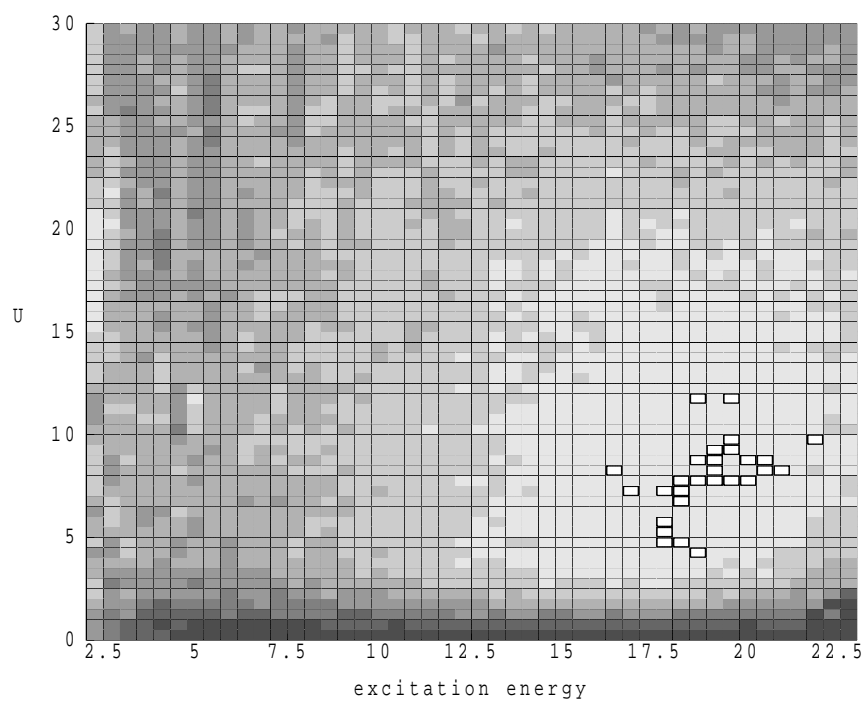
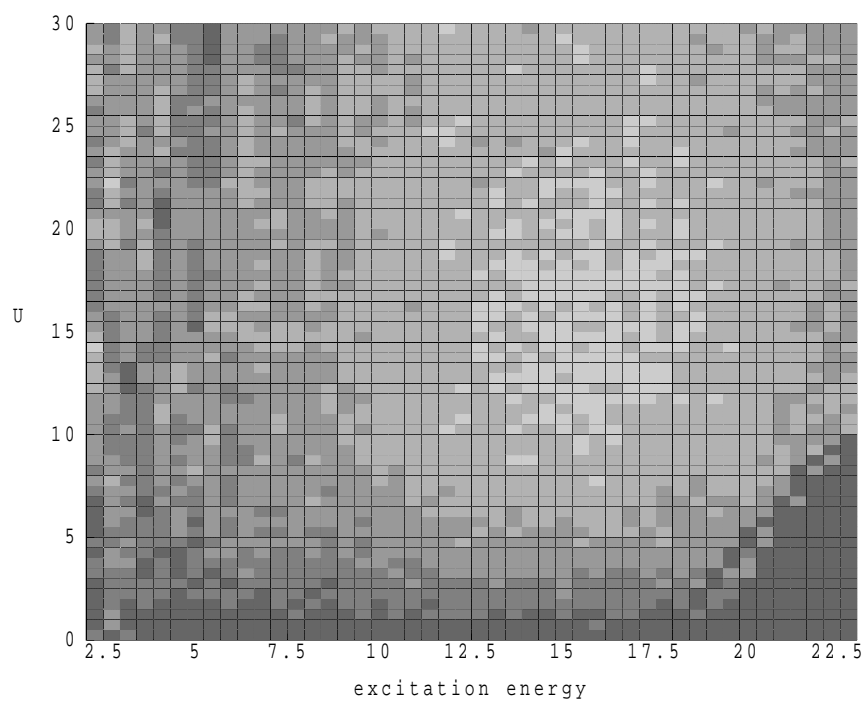


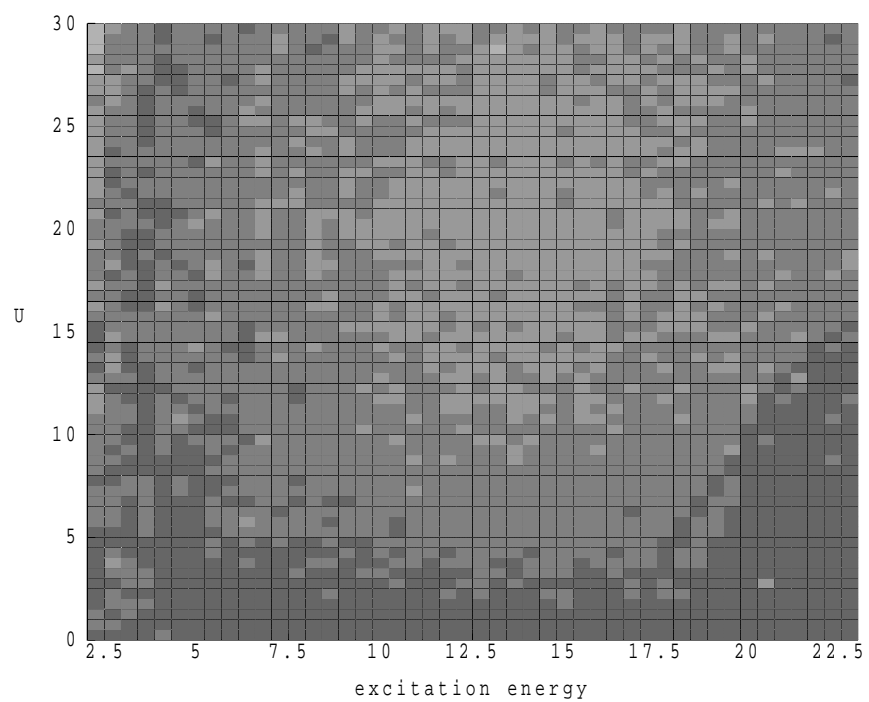
FIG. 1. The values of  $\gamma$  for the spacing between the many particle ground-state and the first excited state as function of the disorder  $W$  for a  $4 \times 3$  lattice with 4 electrons. Inset: the distribution  $P(s)$  for different values of disorder and a given interaction ( $U = 10V$ )



(a)



(b)



(c)

SHADING KEY			
	0 . 1 2 5		0 . 6 2 5
	0 . 2 5		0 . 7 5
	0 . 3 7 5		0 . 8 7 5
	0 . 5		1

FIG. 2. A gray scale map of the values of  $\gamma$  as function of the interaction strength  $U$  and excitation energy in units of the single electron level spacing  $\Delta$  for a  $4 \times 3$  lattice with 4 electrons (a)  $W = 10V$ , (b)  $W = 20V$ , (c)  $W = 30V$ . The corresponding single level spacings are (a)  $\Delta = 0.899V$ , (b)  $\Delta = 1.595V$ , and (c)  $\Delta = 2.344V$ .

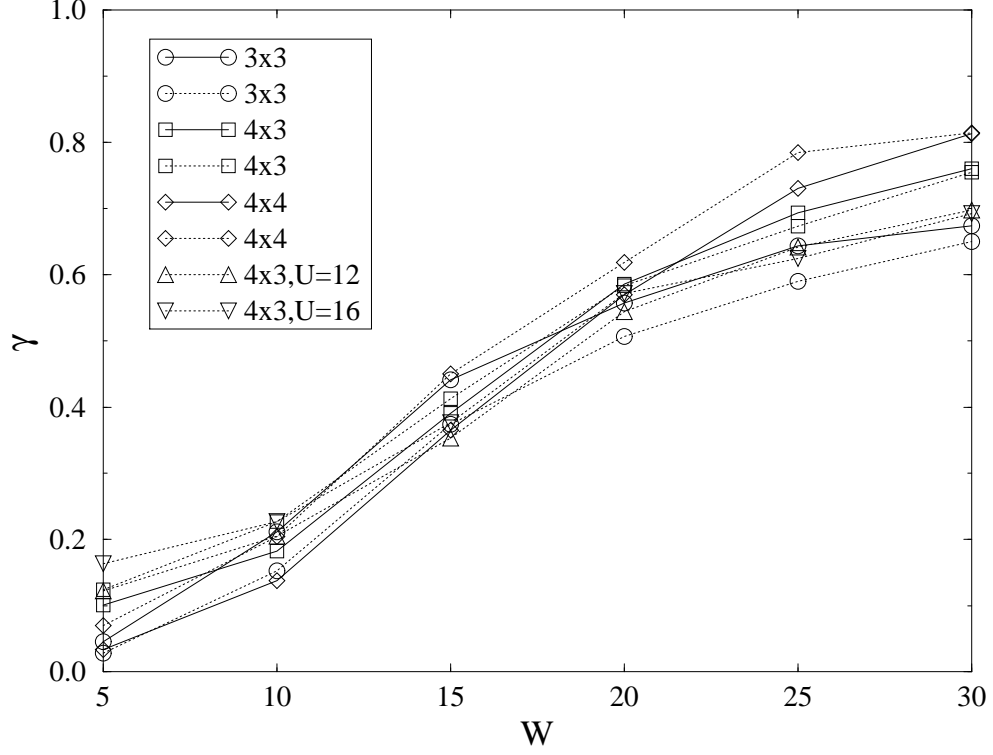


FIG. 3. The values of  $\gamma$  as function of disorder  $W$  for a  $3 \times 3$  lattice with 3 electrons,  $4 \times 3$  lattice with 4 electrons,  $4 \times 4$  lattice with 4 electrons, at interaction strength  $U = 8V$ , and for a  $4 \times 3$  lattice with 4 electrons also for  $U = 12V$  and  $U = 16V$ . The full line corresponds to the value of  $\gamma$  averaged over the lowest 3% of the spacings in the many particle spectrum, and the dotted line corresponds to  $\gamma$  averaged over the lowest 10% of the spacings.